THE FLUIDITY OLYMPICS

Jinfeng Liao

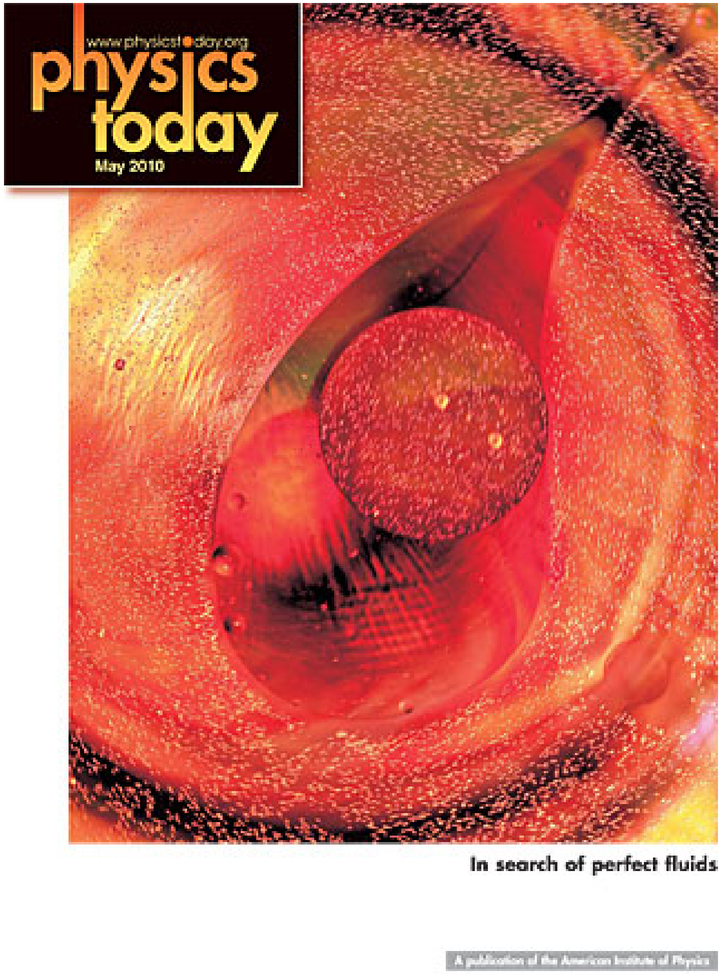
Lawrence Berkeley National Laboratory

OUTLINE

- Brief introduction: length scales in fluids
- An intrinsic fluidity measure: winner?
- Supercritical liquids: supercritical QGP?
- Applicability of hydro for heavy ion collisions: expectation for LHC
- Summary

JL & Koch, Phys.Rev.C, 81, 014902(2010),arXiv:0909.3105[hep-ph]

IN SEARCH OF PERFECT FLUIDS


www.physicstoday.org

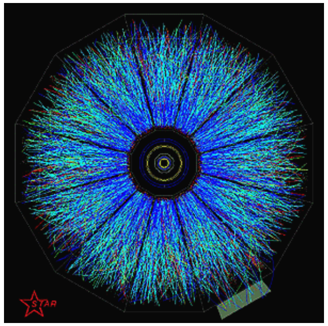
physics today

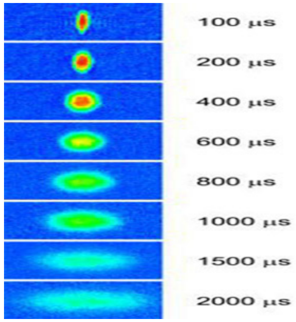
May 2010

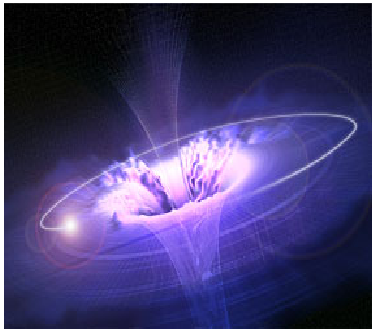
In search of perfect fluids

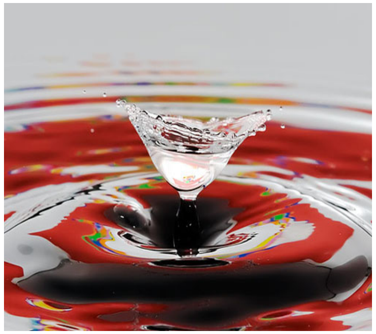
A publication of the American Institute of Physics

players



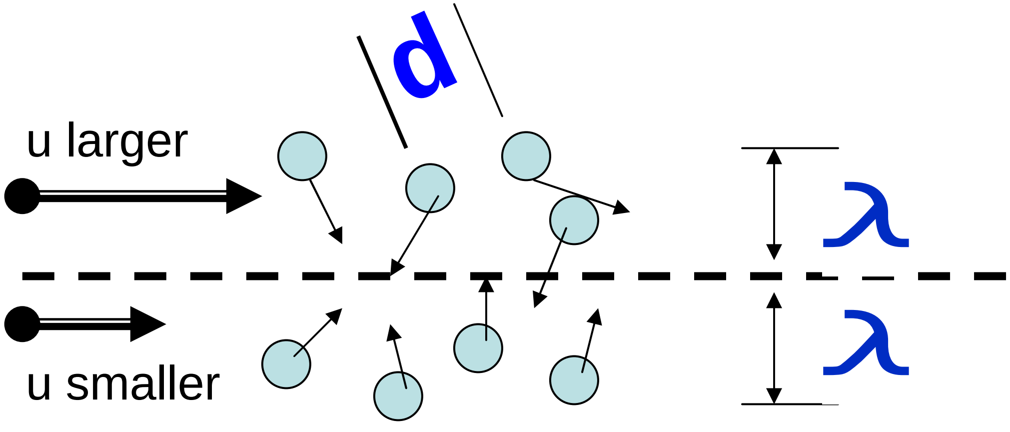


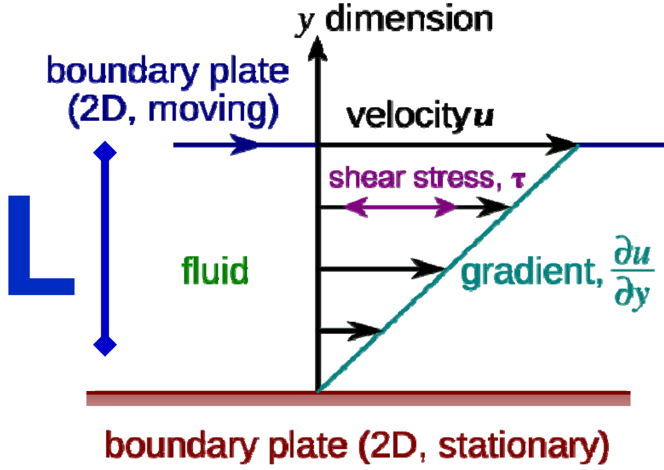




SCALES !!!

Length Scales in Fluids





Fluidity as an intrinsic property:

- the typical transport scale
  → e.g. mean free path
- an **inherent** yardstick:
  → e.g. inter-particle distance

$$\frac{\lambda}{d}$$

Applicability of hydro:

- the typical transport scale
  → e.g. mean free path
- the **exterior** setting:
  → scale for flow field variance

$$\frac{\lambda}{L}$$

shear viscosity : $\eta \sim \rho v_T \lambda \sim n p_T \lambda \sim p_T / \sigma$

WHAT HYDRO EQNS. SAY

Newton's 2nd Law: **Acceleration = $\frac{\text{Force}}{\text{Inertia}}$**

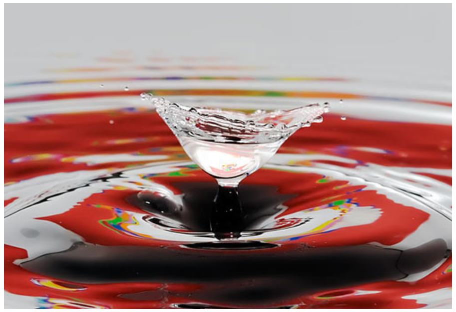
Relativistic

$$\gamma^2(\partial_t + \vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{w/c^2}\left(\vec{\nabla}p + \frac{\vec{v}}{c}\partial_0 p\right) + \frac{\eta}{w/c^2}\partial_\nu\vec{\Sigma}^{\nu i}$$

Fluid acceleration **Driving force / Inertia** **Friction / Inertia**

Non-Relativistic

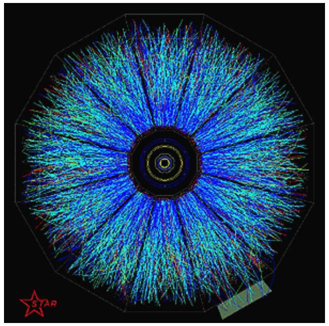
$$(\partial_t + \vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p}{\rho} + \frac{\eta}{\rho}\nabla_j\vec{\Sigma}^{ji}$$



η[water] ~ 10 * η[vapor]

$\frac{\eta}{\rho}$[water] ~ 0.01 * $\frac{\eta}{\rho}$[vapor]

RHIC MARRIAGE WITH ETA/S



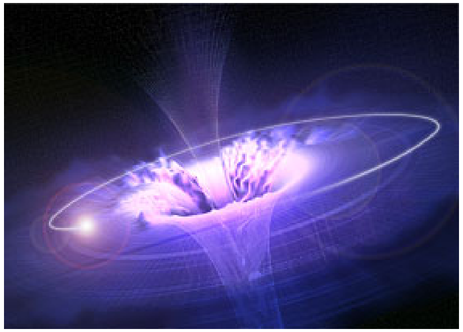
$$\frac{\eta}{s}$$

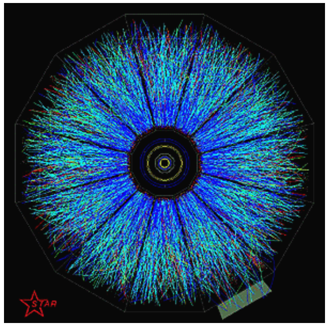
Dated back to Gyulassy&Danielewicz 1984

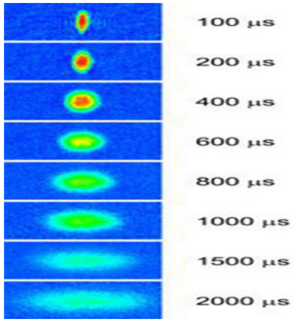
$$\frac{\eta}{w\approx Ts}\times\frac{1}{\tau}=\frac{\eta}{s}\times\frac{1}{T\tau}<<1$$

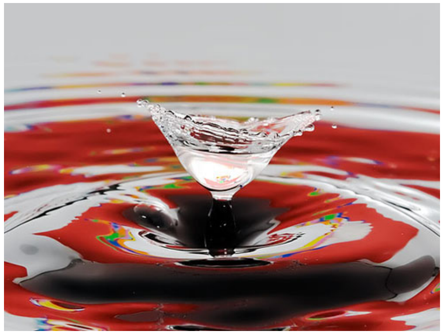
Comment: s comes from w, the inertia

The eta/s Triumph









$$\frac{\eta}{s}[\text{AdS BH}] \lesssim \frac{\eta}{s}[\text{sQGP , cold atom}] \ll \frac{\eta}{s}[\text{water}]$$

FLUID INERTIA: FROM R TO NR

- ☐ **Fluid inertia in hydro equations:**

Relativistic

$$\gamma^2(\partial_t + \vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{w/c^2}\left(\vec{\nabla}p + \frac{\vec{v}}{c}\partial_0 p\right) + \frac{\eta}{w/c^2}\partial_\nu \vec{\Sigma}^{\nu i}$$

Non-Relativistic

$$(\partial_t + \vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p}{\rho} + \frac{\eta}{\rho}\nabla_j \vec{\Sigma}^{ji}$$

- ☐ **Thermodynamics:**

$$w_{\mathrm{R}} = \epsilon + p = Ts + \mu_{\mathrm{R}} n,$$

$$w = Ts + (\mu_{\mathrm{NR}} + m)n \overset{T \ll m}{\longrightarrow} mn \equiv \rho$$

- ☐ **When entropy density dominates inertia?**

$$\boxed{w \overset{T \gg \mu_{\mathrm{R}}}{\longrightarrow} Ts}$$

- ☐ **What is the original KSS conjecture?**

$$\frac{\eta}{s} \geqslant \frac{\hbar}{4\pi k_B}$$

for all relativistic quantum field theories at finite temperature and zero chemical potential.

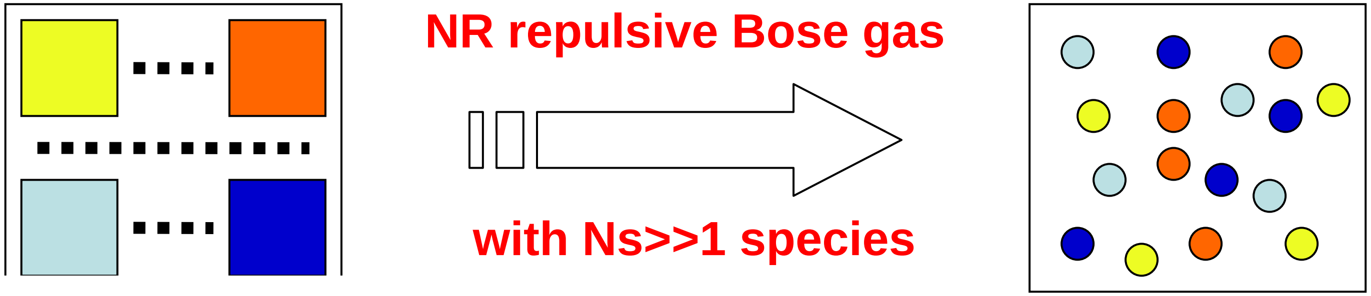
See also critical evaluation of several variants with great details by Cohen, et al

Ts is NOT everywhere the measure of inertia; need new fluidity measure !

THE ETA/S PITFALL

A counter-example:

from T. Cohen, et al *[Phys.Rev.Lett.99,021602(2007); JHEP0802:026,2008]*



NR repulsive Bose gas

with Ns>>1 species

$d \gg \lambda_{T\text{-}dB} \gg a_0$

weakly coupled classical gas:

very viscous

$$\rho c^2 >> w = \varepsilon + p = T \cdot s + \mu \cdot n$$

$$s \propto \log(N_s)$$

$$\mu \propto -\log(N_s)$$

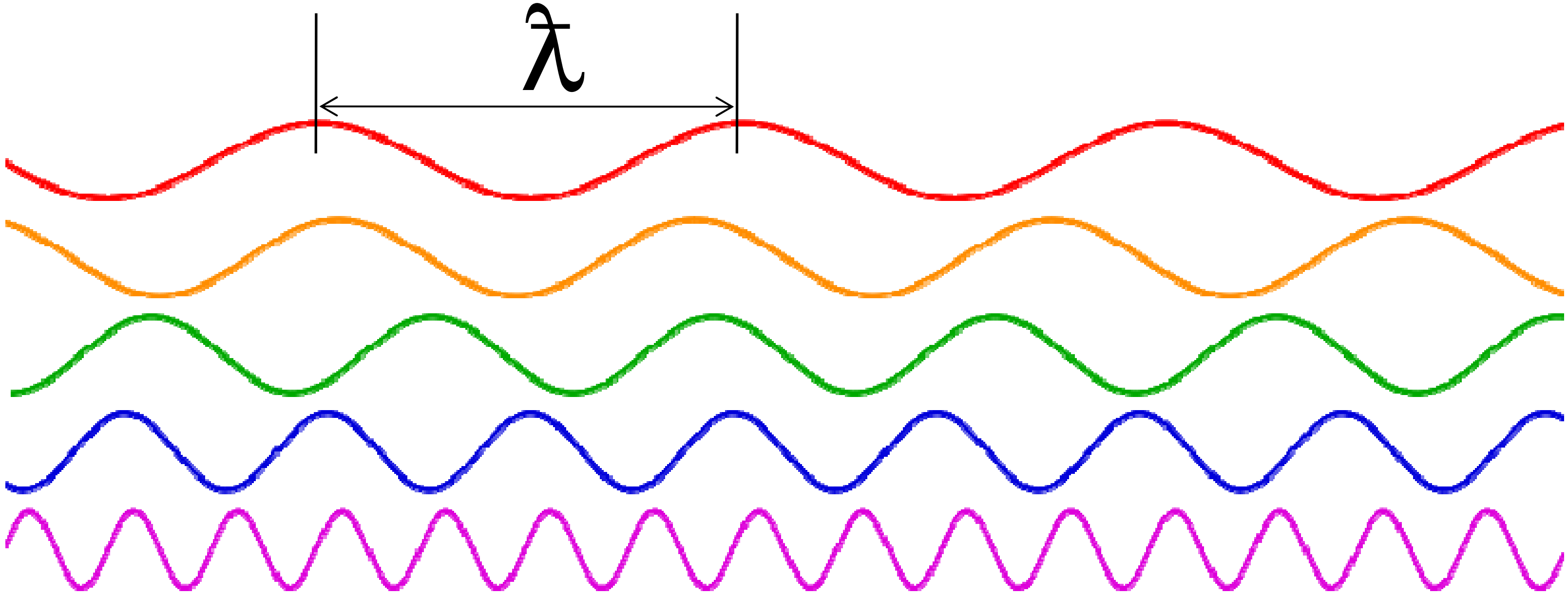
$$\frac{\eta}{s} = \frac{C_{\rm hs}\,\xi^3\sqrt{mT_0}}{a^2 n_0\left[\log\left(\frac{(mT_0)^{3/2}}{n_0}\right) + \frac{5}{2} + \log(\xi) + \log(N_s)\right]}.$$

$$\frac{\eta}{s} \sim \frac{1}{\xi} \rightarrow 0?$$! Gibbs mixing entropy

The eta/s FAILS to reflect the fluidity!

PROBE THE TRANSPORT SCALE

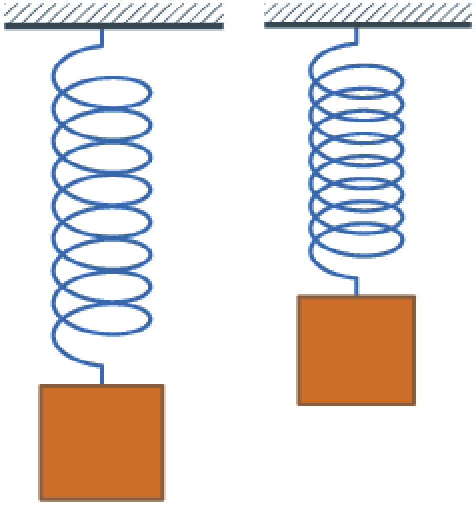
Probe a fluid with sound wave,
Starting from the **VERY** long wavelength, i.e.: *the long wavelength limit*
And, gradually reduce the wavelength…

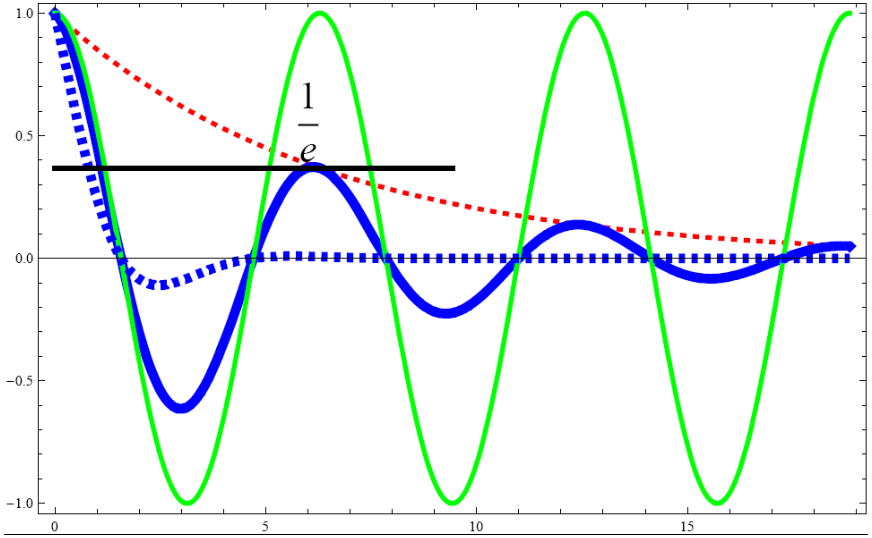


At some probe scale (wavelength): expect breakdown of hydro
→That scale is an intrinsic, transport scale of the fluid
→→ the scale we want to pinpoint

SOUND ATTENUATION IN VISCOUS FLUID

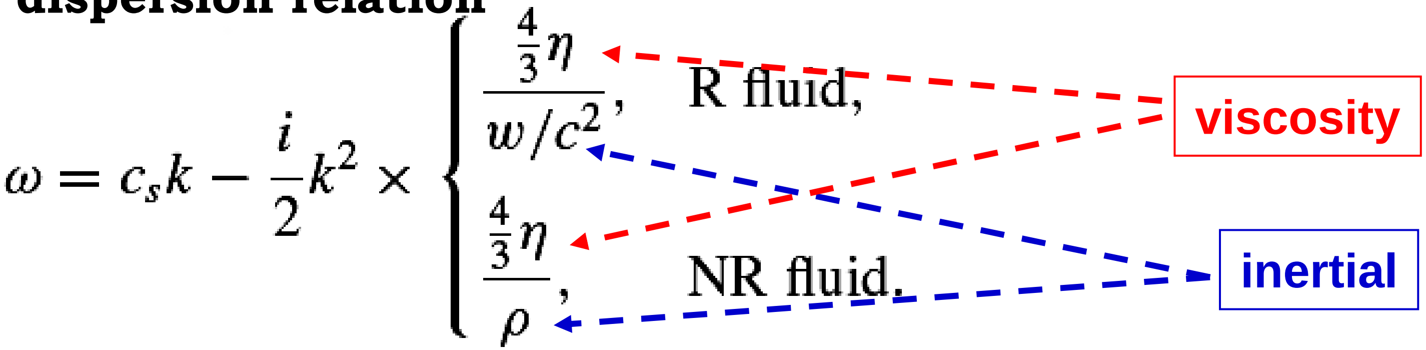
- ☐ **Sound wave probes fluid dissipation like a harmonic oscillator**





(may add bulk viscosity and thermal conductivity)

Sound dispersion relation



$$\omega = c_s k - \frac{i}{2}k^2 \times \begin{cases} \dfrac{\frac{4}{3}\eta}{w/c^2}, & \text{R fluid,} \\ \dfrac{\frac{4}{3}\eta}{\rho}, & \text{NR fluid.} \end{cases}$$

viscosity

inertial

THE WAVE-LESS-LENGTH

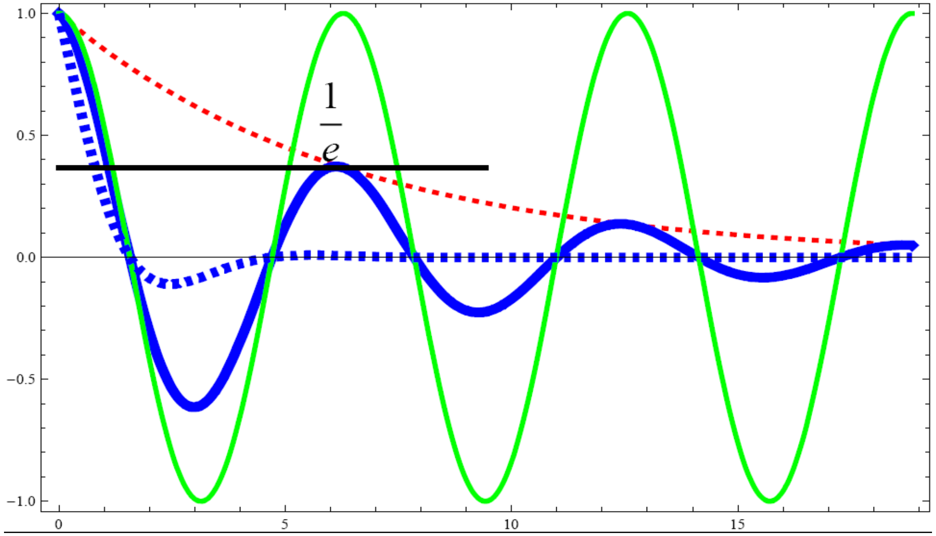
☐ **When a sound wave ceases to propagate …**

$$\left|\frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\omega}\right| \ll 1 \rightarrow$$

Wave-length

$$\lambda_s = \frac{2\pi}{k} \gg \frac{4\pi}{3} L_\eta$$

Wave-less-length

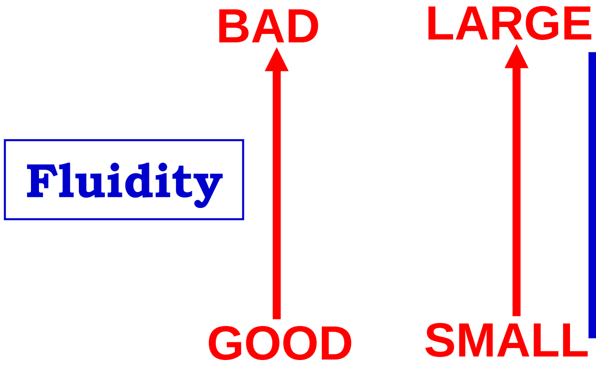


$$L_\eta \equiv \begin{cases} \dfrac{\eta}{(w/c^2)c_s}, & \text{R fluid,} \\ \dfrac{\eta}{\rho c_s}, & \text{NR fluid.} \end{cases}$$

- ☐ ***well defined*** (in opposite to M.F.P.) : all measurable/calculable
- ☐ sets the ***"Long Wavelength Limit"***
- ☐ reduces to ***M.F.P.*** at kinetic limit

$$\eta \sim \rho v_T l_{\mathrm{MFP}} \qquad c_s = \sqrt{\frac{\partial P}{\partial \rho}} \sim \sqrt{\frac{k_B T}{M}} \sim v_T$$

"QUALITY MEASURE" OF FLUIDITY



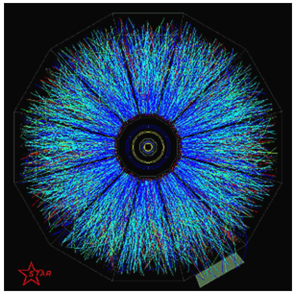
Fluidity

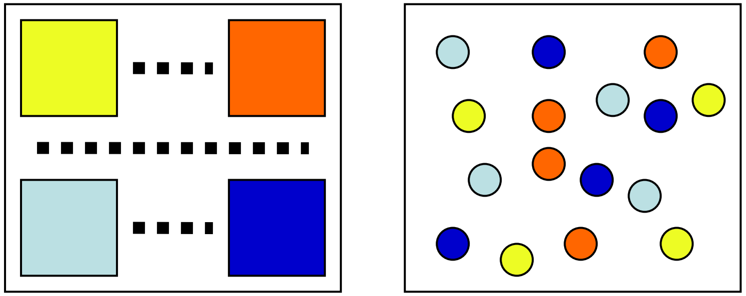
$$\boxed{\mathcal{F} \equiv \frac{L_\eta}{L_n}}$$

$$L_\eta \equiv \begin{cases} \dfrac{\eta}{(w/c^2)c_s}, & \text{R fluid,} \\ \dfrac{\eta}{\rho c_s}, & \text{NR fluid.} \end{cases}$$

$$L_n \equiv \frac{1}{n^{\frac{1}{3}}} \qquad < w(\vec{x}) \cdot w(0) >$$

Micro. short range order





$w \sim Ts$

$c_s \sim 0.5$

$L_n \sim 1/T$

$\Rightarrow \underline{\mathcal{F} \sim \eta/s < 1}$

$\eta \sim (mT)^{1/2} / a^2 \sim \xi^{-1}$

$\rho \sim n\,m \sim \xi^{-4}$

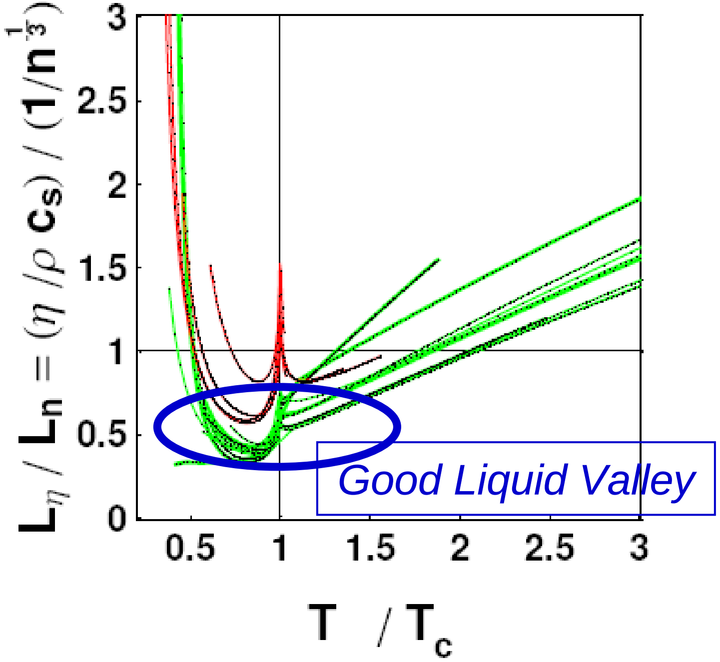
$c_s \sim (T/m)^{1/2} \sim \xi^{-1}$

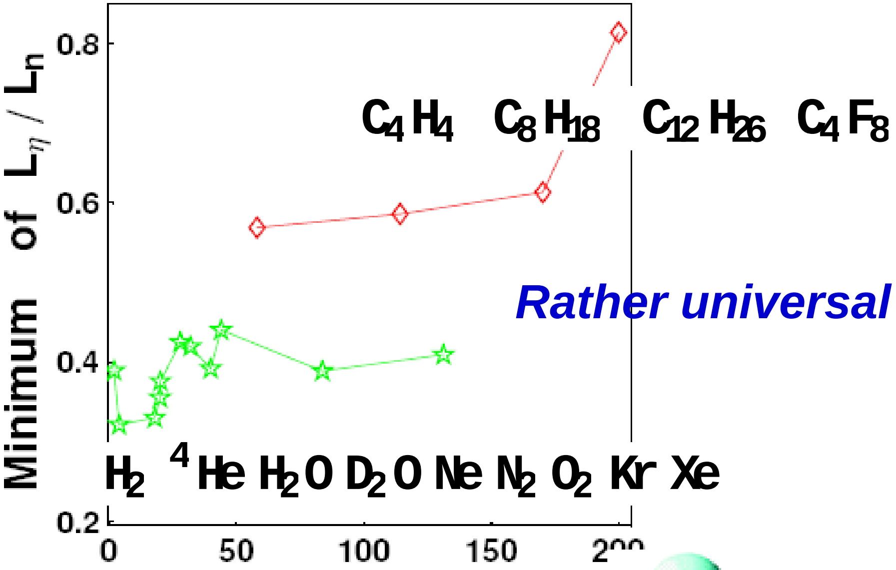
$L_n \sim n^{-1/3} \sim \xi^{4/3}$

$\Rightarrow \underline{\mathcal{F} \sim \xi^{8/3} \gg 1}$

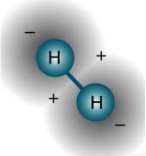
Application: Critical Fluid

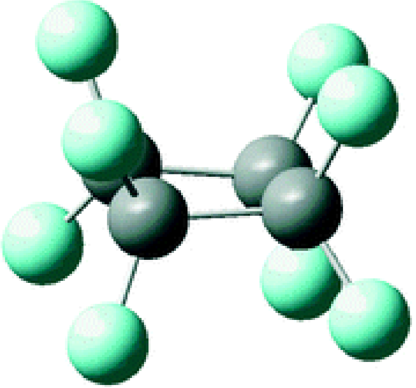
- ☐ **16 fluids:** molar mass,Tc,Pc spanning 2 orders of magnitude
- ☐ **critical fluid: P=Pc** [data from NIST]





A good liquid
is a good liquid !





Critical Behavior

$$\xi \to \infty$$

$$\eta \sim \xi^{x_\eta}$$

$$c_s \to \xi^{-\gamma/2\nu}$$

$$\left(\frac{\eta}{\rho\, c_s}\right) \Big/ \xi \to \xi^{x_\eta+\gamma/2\nu-1}$$

Mean field:

$$x_\eta = 0$$

$$\gamma = 1$$

$$\nu = 1/2$$

$$\left(\frac{\eta}{\rho\, c_s}\right) \Big/ \xi \to \xi^{0}$$

Epsilon expansion:

$$x_\eta = 0.065$$

$$\gamma = 1.167$$

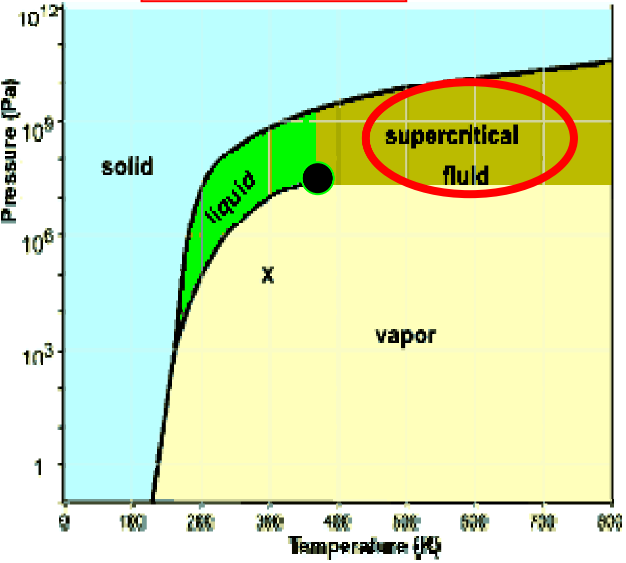
$$\nu = 0.583$$

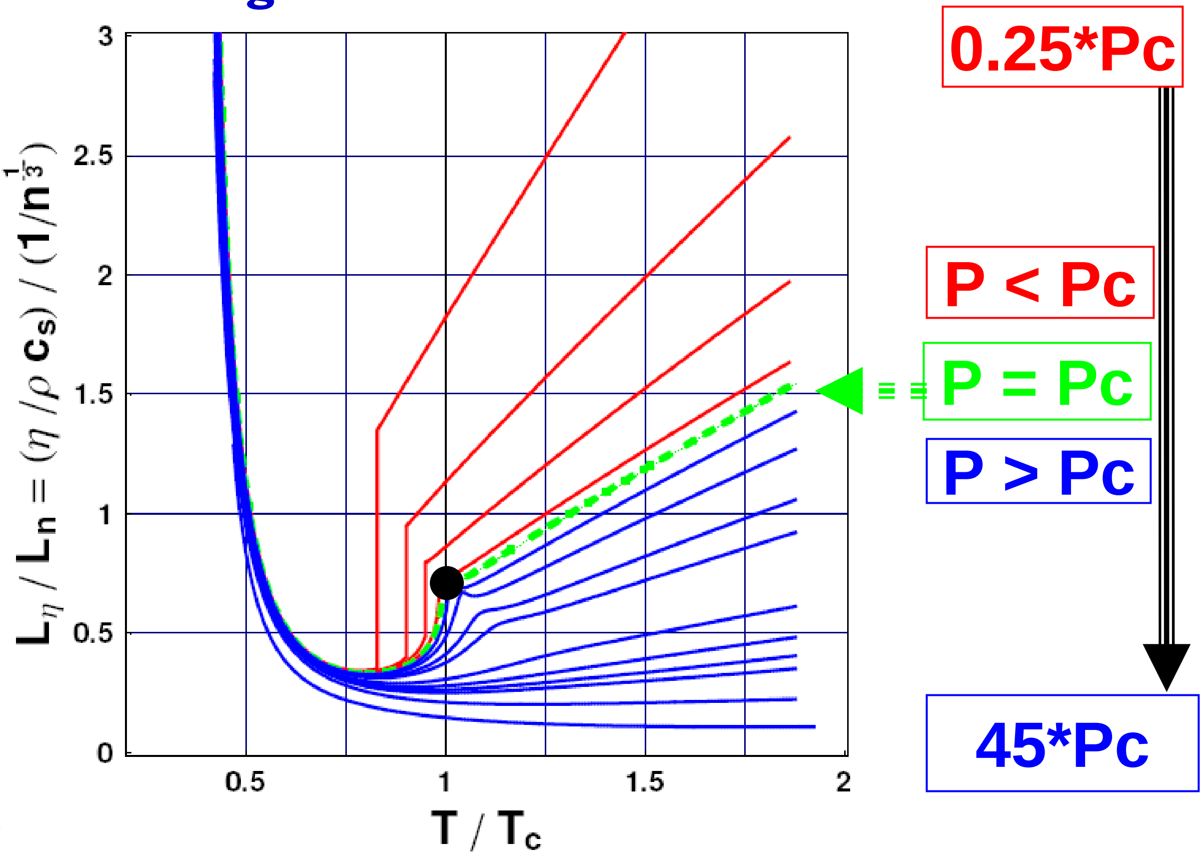
$$\left(\frac{\eta}{\rho\, c_s}\right) \Big/ \xi \to \xi^{0.065}$$

SUPERCRITICAL FLUID

Supercritical fluid: T>Tc & P>Pc region

Water P-T





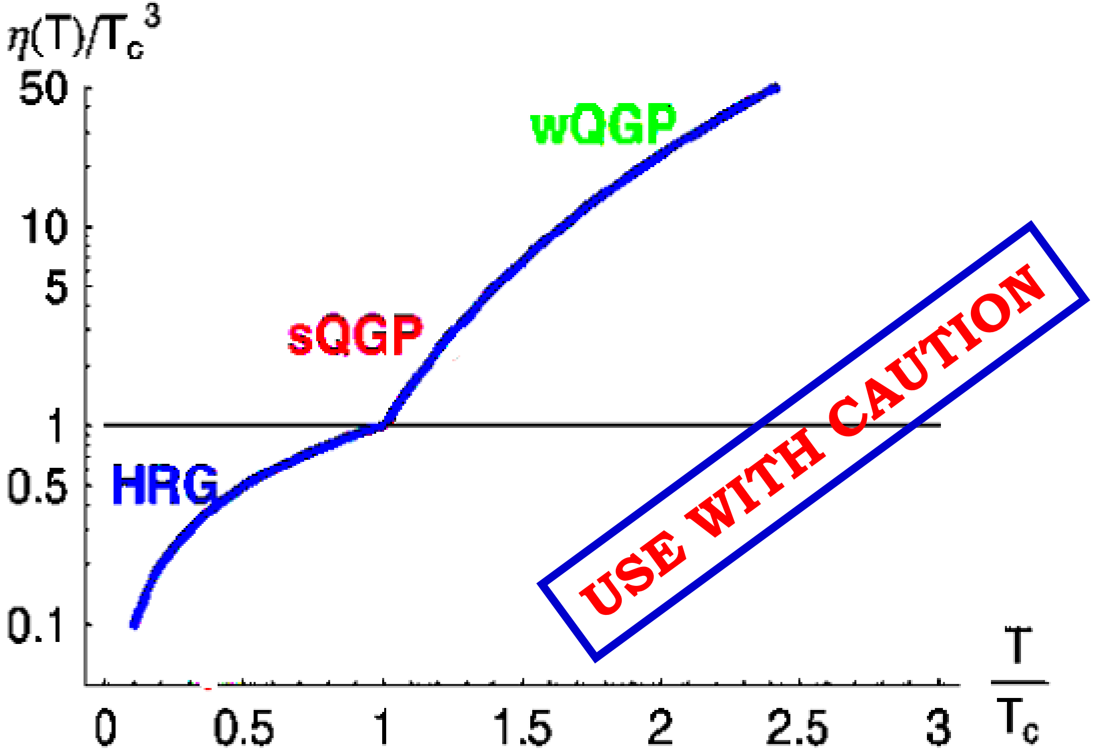
Used in dry cleaning, decaffeinating coffee,……

☐ The best fluidity for a substance is **NOT** around the critical point.

☐ The fluidity gets dramatically **enhanced** in the supercritical region!

ETA: THE QCD MATTER

Hirano & Gyulassy, Nucl.Phys.A769, 71(2006).



USE WITH CAUTION

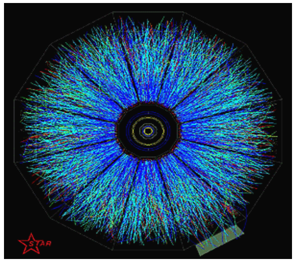
$$\eta(T) \approx T_c^3 \begin{cases} (T/T_c)^1, & T < T_c, \\ (T/T_c)^3[1+w(T)\ln(T/T_c)]^2, & T > T_c. \end{cases}$$

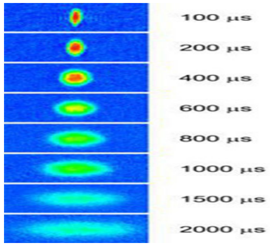
HRG:
Hadron resonance gas
(Prakash, Venugopalan, et al;
Demir & Bass;
Noronha, Greiner et al;
ChPT by Fernandez-Fraile &
Nicola;……)

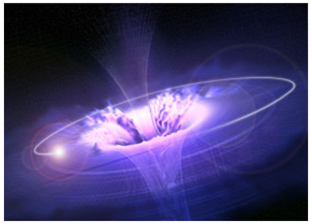
sQGP:
Strongly-coupled QGP
(Molecular Dynamics,
by Gelman, Shuryak, Zahed,
Liao, Dusling, Cho, et al;
gluon trasnport model
by Xu, Greiner et al;
AdS/CFT by Son, et al)

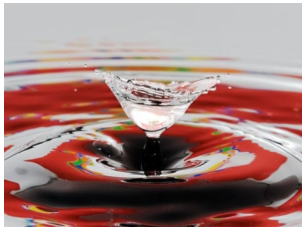
wQGP:
Weakly-coupled QGP
(at high T by Arnold-Moore-Yaffe)

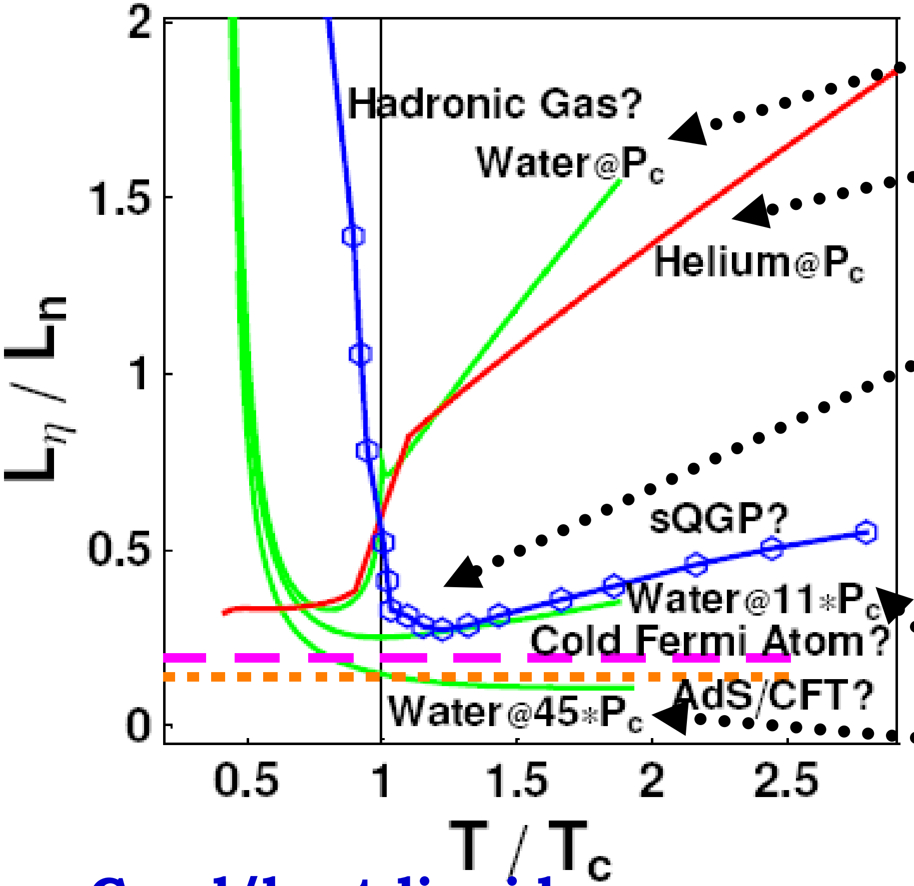
FLUIDITY OLYMPICS











critical water

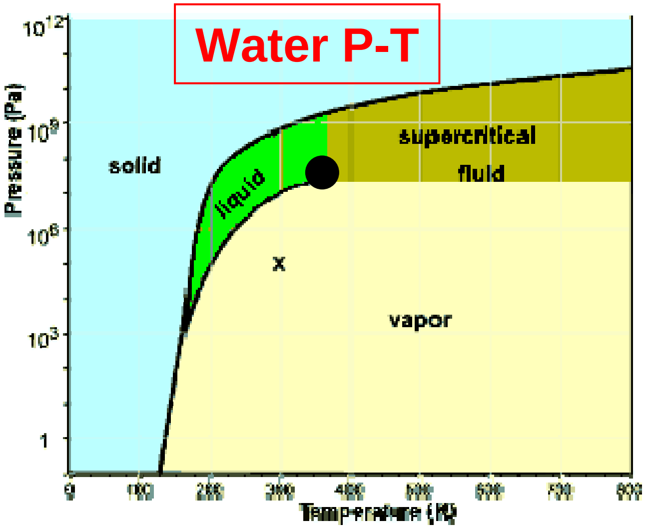
critical helium

QCD matter
(estimate):
eta from the parameterization;
thermodynamics from lattice QCD.

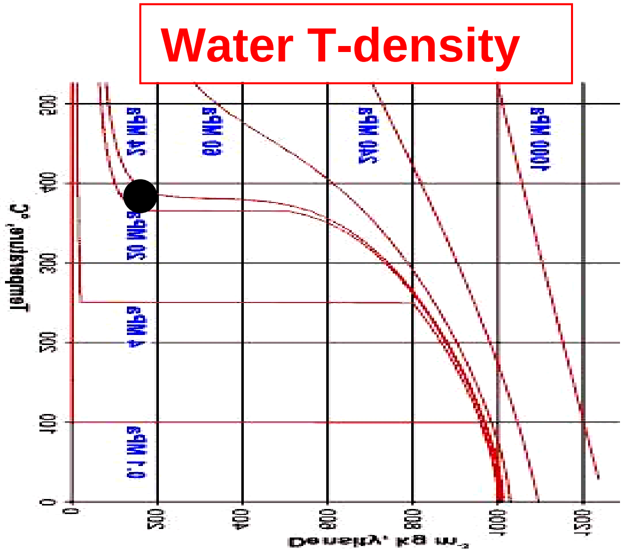
supercritical water

- **Good/best liquid:**
  transport scale approaches micro scale;
- ***The sQGP reaches such a situation.***

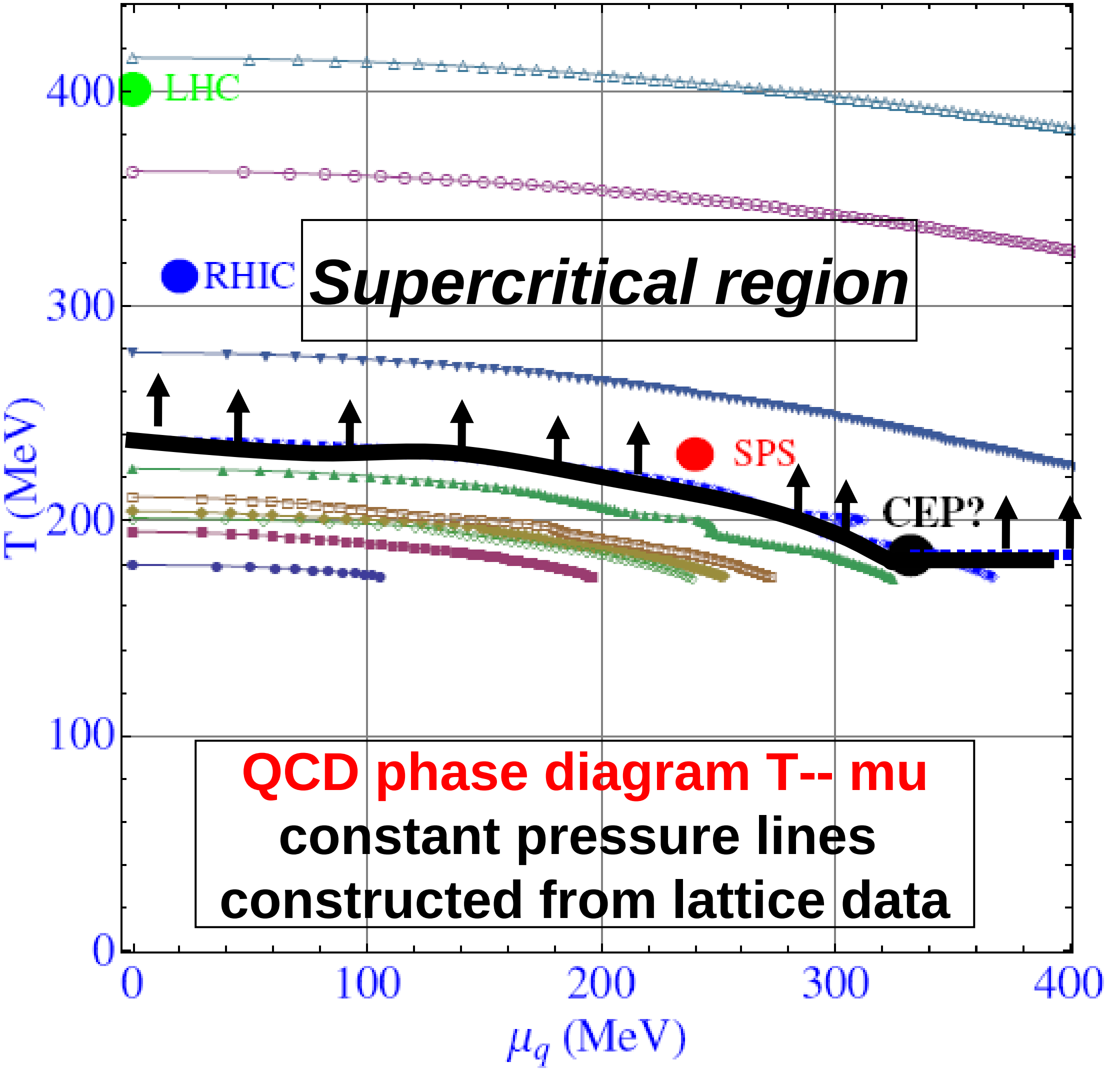
SQGP AS SUPERCRITICAL QGP?



Water P-T



Water T-density



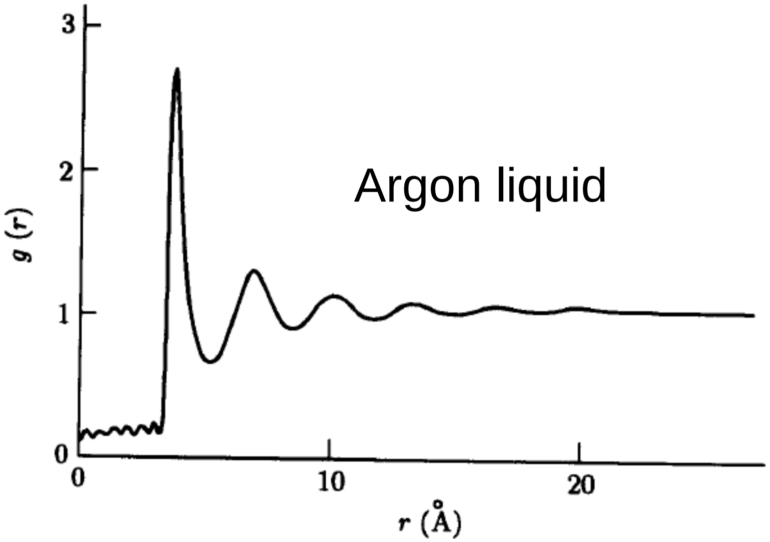
Supercritical region

QCD phase diagram T-- mu

constant pressure lines

constructed from lattice data

WHAT IS SO SPECIAL?

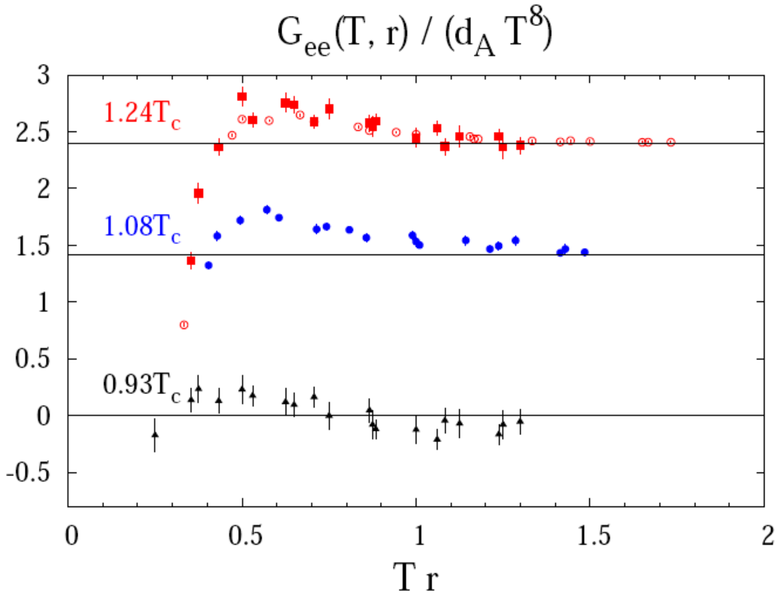


Argon liquid

- Solid: many more profound peaks (phonons travel far)
- Gas: trivial (particles travel far)
- Liquid: in between

(guess) sLiquid: only ~1 peaks
(really particle/phonon all stuck)

QGP near Tc
(Meyer, 2008)

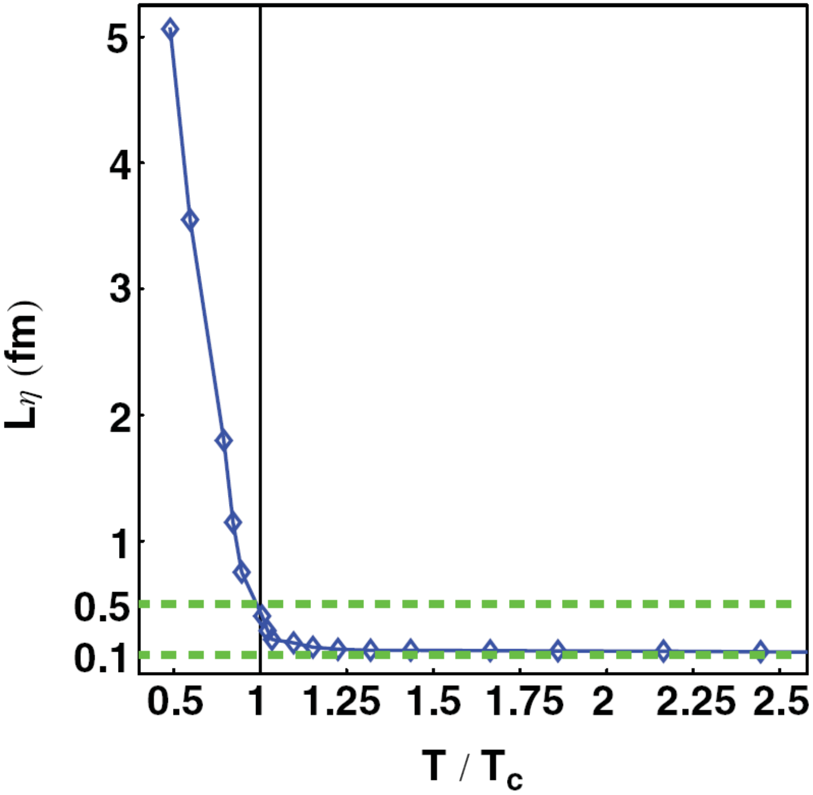


Also seen in the magnetic component
(D'Alessandro & D'elia, 2007)
(JL & Shuryak, 2008)

APPLICABILITY OF HYDRO

L_η v.s. L_s system size

$\frac{L_\eta}{L_s}$ effective Knudsen #



* Coarse-grain issue:
Fluid cell size > Leta
→ how to take care of the growing Leta in practical implementation?

* Strongly coupled:
2-body or only collectively?

$$\Gamma = \frac{\alpha * n^{1/3}}{T} : \text{liquid}, \sim 1-10$$

But, still possible to infer 2-body:

$$L_\eta \sim \frac{1}{n\sigma} \rightarrow \sigma$$

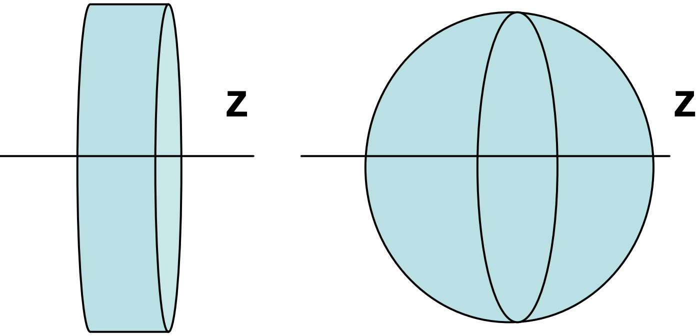
(Molnar & Gyulassy,2001)

GRADIENT EXPANSION IN

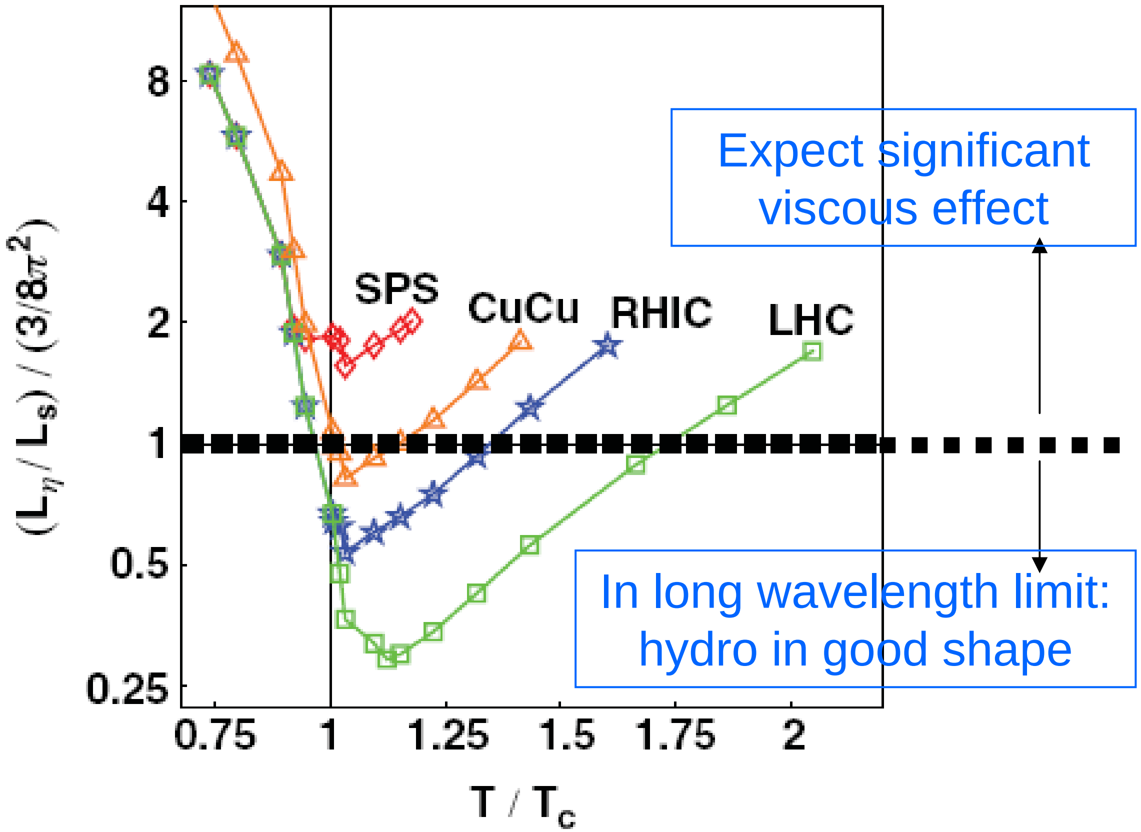
L_η v.s. L_s system size

$\frac{L_\eta}{L_s}$ effective Knudsen #

early time : $L_s \sim 2\,\tau$



late time : $L_s \sim 2\,R_A$



Expect significant viscous effect

In long wavelength limit: hydro in good shape

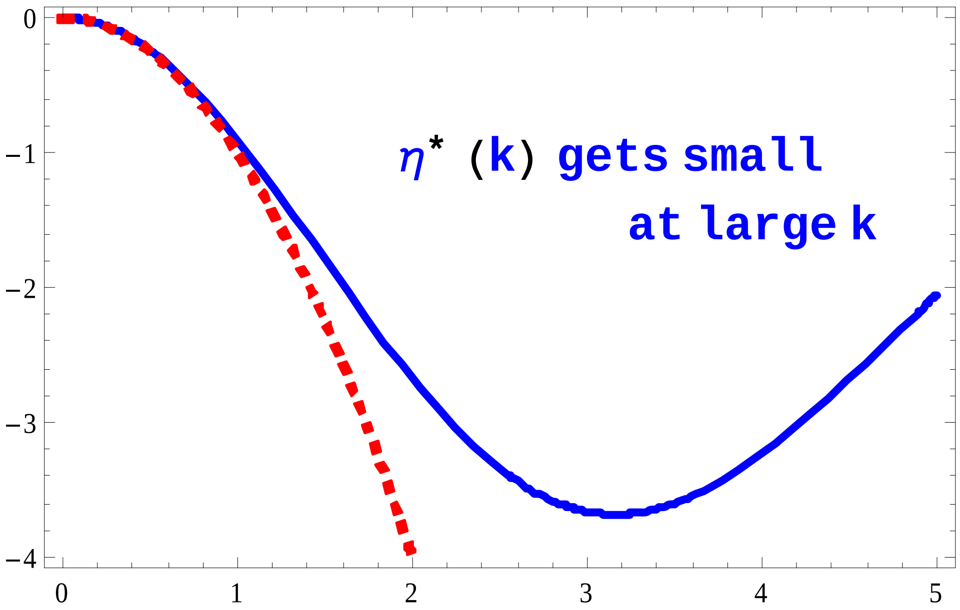
Expect ultimate paradise for hydro at LHC?!

THE SHORT WAVELENGTH END

Hydro: long wavelength limit ;
In heavy ion collisions: not always there
Do we know the short wavelength end?
For a strongly coupled plasma?

Im[ω]

AdS/CFT results
(Kovtun & Starinets,2004)



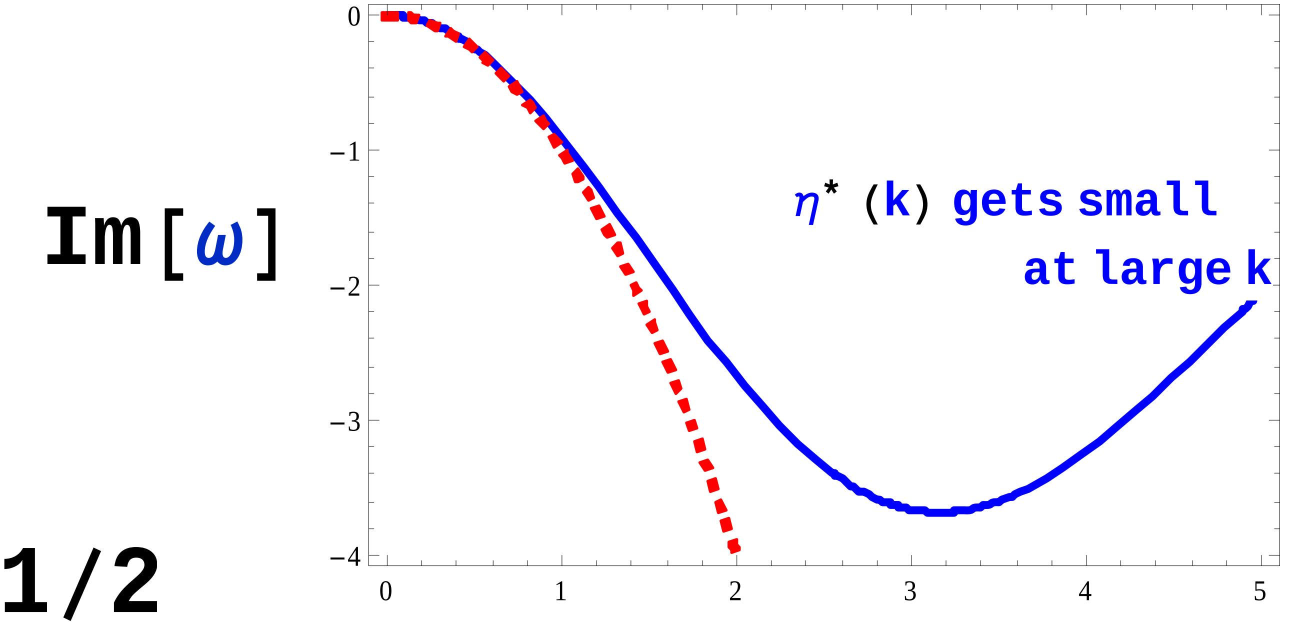
η^* (k) gets small
at large k

$k \sim \frac{1}{L}$

(Lublinski & Shuryak, 2007)
Going to higher order of gradient expansion:
NOT necessarily generating more entropy → $+,-,+,-,+,-,\ldots$

EXPLORING THE SHORT

At short wavelength end: interesting physics as well !
→ Dialing the energy and nuclei size to probe it !

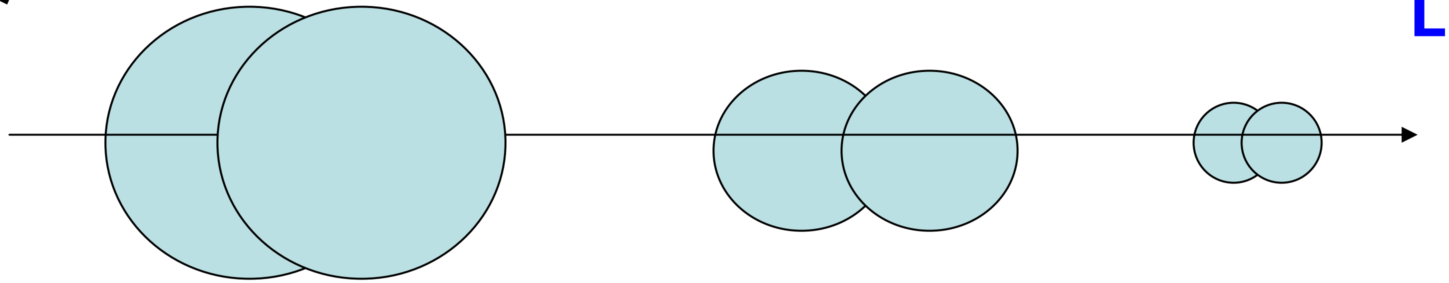


η^* (k) gets small at large k

$k \sim \frac{1}{L}$

$S^{1/2}$

A



SUMMARY

- The important length scale for a fluid that governs the transport and dissipation:

$$L_\eta \equiv \begin{cases} \dfrac{\eta}{(w/c^2)c_s}, & \text{R fluid,} \\ \dfrac{\eta}{\rho c_s}, & \text{NR fluid.} \end{cases}$$

- A new measure of fluidity for comparing fluids from completely different scales

- A good liquid is a good liquid: Leta approaching micro. Scale quite universal → *fluidality* $\mathcal{F} \sim 0.1 - 0.5$

- Supercritical fluid and supercritical QGP

- Better hydro description at LHC

THANK YOU !